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Numerical Analysis (10th Edition)

Chapter 3.1, Problem 5E 8 Bookmarks Show all steps: ON

Problem

Use appropriate Lagrange interpolating polynomials of degrees one, two, and three to approximate each of the following:

a. $f(8.4)$ if $f(8.1)=16.94410$, $f(8.3)=17.56492$, $f(8.6)=18.50515$, $f(8.7)=18.82091$
b. $f(-\frac{1}{3})$ if $f(-0.75)=-0.07181250$, $f(-0.5)=-0.02475000$, $f(-0.25)=0.33493750$, $f(0)=1.10100000$
c. $f(0.25)$ if $f(0.1)=0.62049958$, $f(0.2)=-0.28398668$, $f(0.3)=0.00660095$, $f(0.4)=0.24842440$
d. $f(0.9)$ if $f(0.6)=-0.17694460$, $f(0.7)=0.01375227$, $f(0.8)=0.22363362$, $f(1.0)=0.65809197$

Step-by-step solution

Step 1 of 11

To construct interpolation polynomial of degree two consider all the three nodal points

i.e. $x_0=8.3,\ x_1=8.6$ and $x_2=8.7$

Then $y_0=f(8.3)=17.56492, y_1=f(8.6)=18.50515$ and $y_2=f(8.7)=18.82091$

We first determine the coefficient polynomial $L_0(x), L_1(x)$ and $L_2(x)$

$$\begin{aligned}L_0(x)&=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\\&=\frac{(x-8.6)(x-8.7)}{(8.3-8.6)(8.3-8.7)}\\&=\frac{x^2-17.3x+74.82}{(-0.3)(-0.4)}\\&=\frac{1}{0.12}(x^2-17.3x+74.82)\end{aligned}$$

$$\begin{aligned}L_1(x)&=\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\\&=\frac{(x-8.3)(x-8.7)}{(8.6-8.3)(8.6-8.7)}\\&=\frac{x^2-17x+72.21}{(0.3)(-0.1)}\\&=-\frac{1}{0.03}(x^2-17x+72.21)\end{aligned}$$

and

$$\begin{aligned}L_2(x)&=\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\\&=\frac{(x-8.3)(x-8.6)}{(8.7-8.3)(8.7-8.6)}\\&=\frac{x^2-16.9x+71.38}{(0.4)(0.1)}\\&=\frac{1}{0.04}(x^2-16.9x+71.38)\end{aligned}$$

Now the interpolate polynomial of degree two

$$\begin{aligned}p_2(x)&=L_0(x)\cdot y_0+L_1(x)\cdot y_1+L_2(x)\cdot y_2\\&=\frac{1}{0.12}(x^2-17.3x+74.82)(17.56492)-\frac{1}{0.03}(x^2-17x+72.21)(18.50515)\\&\qquad+\frac{1}{0.04}(x^2-16.9x+71.38)(18.82091)\\&=0.05875x^2+2.14122x-4.25453\end{aligned}$$

And $p_2(8.4)=0.05875(8.4)^2+2.14122(8.4)-4.25453$ by using calculator

= $\boxed{17.87716}$

Comments (5)

Step 2 of 11

To construct interpolation polynomial of degree three consider all the four nodal points

i.e. $x_0=8.3,\ x_1=8.6,\ x_2=8.7$ and $x_3=8.1$

Then $y_0=f(8.3)=17.56492,\ y_1=f(8.6)=18.50515 \quad y_2=f(8.7)=18.82091$

and $y_2=f(8.1)=16.94410$

We first determine the coefficient polynomial $L_0(x), L_1(x), L_2(x)$ and $L_3(x)$

$$\begin{aligned}L_0(x)&=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\\&=\frac{(x-8.6)(x-8.7)(x-8.1)}{(8.3-8.6)(8.3-8.7)(8.3-8.1)}\\&=\frac{(x-8.6)(x-8.7)(x-8.1)}{(-0.3)(-0.4)(0.2)}\\&=\frac{(x-8.6)(x-8.7)(x-8.1)}{0.24}\end{aligned}$$

$$\begin{aligned}L_1(x)&=\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\\&=\frac{(x-8.3)(x-8.7)(x-8.1)}{(8.6-8.3)(8.6-8.7)(8.6-8.1)}\\&=\frac{(x-8.3)(x-8.7)(x-8.1)}{(0.3)(-0.1)(0.5)}\\&=-\frac{(x-8.3)(x-8.7)(x-8.1)}{0.015}\end{aligned}$$

$$\begin{aligned}L_2(x)&=\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}\\&=\frac{(x-8.3)(x-8.6)(x-8.1)}{(8.7-8.3)(8.7-8.6)(8.7-8.1)}\\&=\frac{(x-8.3)(x-8.6)(x-8.1)}{(0.4)(0.1)(0.6)}\\&=\frac{(x-8.3)(x-8.6)(x-8.1)}{0.024}\end{aligned}$$

and

$$\begin{aligned}L_3(x)&=\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\\&=\frac{(x-8.3)(x-8.6)(x-8.7)}{(8.1-8.3)(8.1-8.6)(8.1-8.7)}\\&=\frac{(x-8.3)(x-8.6)(x-8.7)}{(-0.2)(-0.5)(-0.6)}\\&=-\frac{(x-8.3)(x-8.6)(x-8.7)}{0.06}\end{aligned}$$

Now the interpolate polynomial of degree three

$$\begin{aligned}p_3(x)&=L_0(x)\cdot y_0+L_1(x)\cdot y_1+L_2(x)\cdot y_2+L_3(x)\cdot y_3\\&=\frac{(x-8.6)(x-8.7)(x-8.1)}{0.24}(17.56492)-\frac{(x-8.3)(x-8.7)(x-8.1)}{0.015}(18.50515)\\&\qquad+\frac{(x-8.3)(x-8.6)(x-8.1)}{0.024}(18.82091)-\frac{(x-8.3)(x-8.6)(x-8.7)}{0.06}(16.94470)\\&=-0.00208x^3+0.11208x^2+1.68621x-2.96080\end{aligned}$$

And $p_3(8.4)=-0.00208(8.4)^3+0.11208(8.4)^2+1.68621(8.4)-2.96080$ by using calculator

= $\boxed{17.87714}$

Comments (7)

Step 3 of 11

(b) We are given with $f(-0.75)=0.07181250, f(-0.5)=0.0247500, f(-0.25)=0.33493750$ and $f(0)=1.10100000$

To construct interpolation polynomial of degree one, consider only two nodal points

$x_0=-0.5$ and $x_1=-0.25$

Then $y_0=f(-0.5)=0.0247500$, and $y_1=f(-0.25)=0.33493750$

We first determine the coefficient polynomial $L_0(x)$ and $L_1(x)$

$$\begin{aligned}L_0(x)&=\frac{x-x_1}{x_0-x_1}\\&=\frac{x-(-0.25)}{-0.5-(-0.25)}\\&=\frac{1}{-0.5+0.25}(x+0.25)\\&=\frac{x+0.25}{-0.25}\end{aligned}$$

$$\begin{aligned}L_1(x)&=\frac{x-x_0}{x_1-x_0}\\&=\frac{x-(-0.5)}{-0.25-(-0.5)}\\&=\frac{1}{-0.25+0.5}(x+0.5)\\&=\frac{x+0.5}{0.25}\end{aligned}$$

Now the interpolate polynomial of degree one

$$\begin{aligned}p_1(x)&=L_0(x)\cdot y_0+L_1(x)\cdot y_1\\&=\frac{x+0.25}{-0.25}(-0.02475000)+\frac{x+0.5}{0.25}(0.33493750)\\&=1.43875000x-0.694625000\end{aligned}$$

And $p_1\left(-\frac{1}{3}\right)=1.43875000\left(-\frac{1}{3}\right)-0.694625000$ by using calculator

= $\boxed{0.21504167}$

Comments (1)

Step 4 of 11

To construct interpolation polynomial of degree two consider all the three nodal points

i.e. $x_0=-0.5,\ x_1=-0.25$ and $x_2=0.0$

Then $y_0=f(-0.5)=0.0247500,\ y_1=f(-0.25)=0.33493750$ and $y_2=f(0.0)=1.0100000$

We first determine the coefficient polynomial $L_0(x), L_1(x)$ and $L_2(x)$

$$\begin{aligned}L_0(x)&=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\\&=\frac{(x-(-0.25))(x-0)}{(-0.5-(-0.25))(-0.5-0)}\\&=\frac{x(x+0.25)}{(-0.5+0.25)(-0.5)}\\&=\frac{x(x+0.25)}{(-0.25)(-0.5)}\\&=\frac{x(x+0.25)}{0.125}\end{aligned}$$

$$\begin{aligned}L_1(x)&=\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\\&=\frac{(x-(-0.5))(x-0)}{(-0.25-(-0.5))(-0.25-0)}\\&=\frac{x(x+0.5)}{(-0.25+0.5)(-0.25)}\\&=\frac{x(x+0.5)}{(0.25)(-0.25)}\\&=-\frac{x(x+0.5)}{0.0625}\end{aligned}$$

and

$$\begin{aligned}L_2(x)&=\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\\&=\frac{(x-(-0.5))(x-(-2.5))}{(0-(-0.5))(0-(-2.5))}\\&=\frac{(x+0.5)(x+0.25)}{(0.5)(0.25)}\\&=\frac{(x+0.5)(x+0.25)}{0.125}\end{aligned}$$

Now the interpolate polynomial of degree two

$$\begin{aligned}p_2(x)&=L_0(x)\cdot y_0+L_1(x)\cdot y_1+L_2(x)\cdot y_2\\&=\frac{x(x+0.25)}{0.125}(-0.02475000)-\frac{x(x+0.5)}{0.0625}(0.33493750)\\&\qquad+\frac{(x+0.5)(x+0.25)}{0.125}(1.10100000)\\&=3.25100000x^2+3.87700000x+1.10100000\end{aligned}$$

And $p_2\left(-\frac{1}{3}\right)=3.25100000\left(-\frac{1}{3}\right)^2+3.87700000\left(-\frac{1}{3}\right)+1.10100000$ by using calculator

= $\boxed{0.16988889}$

Comment

Step 5 of 11

To construct interpolation polynomial of degree three consider all the four nodal points

i.e. $x_0=-0.5,\ x_1=-0.25,\ x_2=0.0$ and $x_3=-0.75$

Then $y_0=f(-0.5)=0.0247500,\ y_1=f(-0.25)=0.33493750,\ y_2=f(0.0)=1.0100000$

and $y_3=f(-0.75)=-0.07181250$

We first determine the coefficient polynomial $L_0(x), L_1(x)$ and $L_2(x)$

$$\begin{aligned}L_0(x)&=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\\&=\frac{(x-(-0.25))(x-0)(x-(-0.75))}{(-0.5-(-0.25))(-0.5-0)(-0.5-(-0.75))}\\&=\frac{x(x+0.25)(x+0.75)}{(-0.5+0.25)(-0.5)(-0.5+0.75)}\\&=\frac{x(x+0.25)(x+0.75)}{(-0.25)(-0.5)(0.25)}\\&=\frac{x(x+0.25)(x+0.75)}{0.03125}\end{aligned}$$

$$\begin{aligned}L_1(x)&=\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\\&=\frac{(x-(-0.5))(x-0)(x-(-0.75))}{(-0.25-(-0.5))(-0.25-0)(-0.25-(-0.75))}\\&=\frac{x(x+0.5)(x+0.75)}{(-0.25+0.5)(-0.25)(-0.25+0.75)}\\&=\frac{x(x+0.5)(x+0.75)}{(0.25)(-0.25)(0.5)}\\&=-\frac{x(x+0.5)(x+0.75)}{0.03125}\end{aligned}$$

$$\begin{aligned}L_2(x)&=\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}\\&=\frac{(x-(-0.5))(x-(-0.25))(x-(-0.75))}{(0-(-0.5))(0-(-0.25))(0-(-0.75))}\\&=\frac{(x+0.5)(x+0.25)(x+0.75)}{(0.5)(0.25)(-0.75)}\\&=\frac{(x+0.5)(x+0.25)(x+0.75)}{0.09375}\end{aligned}$$

and

$$\begin{aligned}L_3(x)&=\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\\&=\frac{(x-(-0.5))(x-(-2.5))(x-0)}{(-0.75-(-0.5))(-0.75-(-2.5))(-0.75-0)}\\&=\frac{x(x+0.5)(x+0.25)}{(-0.75+0.5)(-0.75+0.25)(-0.75)}\\&=\frac{x(x+0.5)(x+0.25)}{(-0.25)(-0.5)(-0.75)}\\&=\frac{x(x+0.5)(x+0.25)}{-0.09375}\end{aligned}$$

Now the interpolate polynomial of degree three

$$\begin{aligned}p_3(x)&=L_0(x)\cdot y_0+L_1(x)\cdot y_1+L_2(x)\cdot y_2+L_3(x)\cdot y_3\\&=\frac{x(x+0.25)(x+0.75)}{0.03125}(-0.02475000)-\frac{x(x+0.5)(x+0.75)}{0.03125}(0.33493750)\\&\qquad+\frac{(x+0.5)(x+0.25)(x+0.75)}{0.09375}(1.10100000)-\frac{x(x+0.5)(x+0.25)}{0.09375}(0.07181250)\\&=x^3+4x^2+4.002x+1.101\end{aligned}$$

And $p_3\left(-\frac{1}{3}\right)=\left(-\frac{1}{3}\right)^3+4\left(-\frac{1}{3}\right)^2+4.002\left(-\frac{1}{3}\right)+1.101$ by using calculator

= $\boxed{0.17451852}$

Comment

Step 6 of 11

(c) We are given with $f(0.1)=-0.62049958, f(0.2)=-0.28398668, f(0.3)=0.00660095$ and

$f(0.4)=0.24842440$

To construct interpolation polynomial of degree one, consider only two nodal points

$x_0=0.2$ and $x_1=0.3$

Then $y_0=f(0.2)=-0.28398668$ and $y_1=f(0.3)=0.00660095$

We first determine the coefficient polynomial $L_0(x)$ and $L_1(x)$

$$\begin{aligned}L_0(x)&=\frac{x-x_1}{x_0-x_1}\\&=\frac{x-0.3}{0.2-0.3}\\&=-\frac{1}{0.1}(x-0.3)\end{aligned}$$

$$\begin{aligned}L_1(x)&=\frac{x-x_0}{x_1-x_0}\\&=\frac{x-0.2}{0.3-0.2}\\&=\frac{1}{0.1}(x-0.2)\end{aligned}$$

Now the interpolate polynomial of degree one

$$\begin{aligned}p_1(x)&=L_0(x)\cdot y_0+L_1(x)\cdot y_1\\&=-\frac{1}{0.1}(x-0.3)(-0.28398668)+\frac{1}{0.1}(x-0.2)(0.00660095)\\&=2.90587630x-0.865161940\end{aligned}$$

And $p_1(0.25)=2.90587630(0.25)-0.865161940$ by using calculator

= $\boxed{-0.13869287}$

Comments (1)

Step 7 of 11

To construct interpolation polynomial of degree two consider all the three nodal points

i.e. $x_0=0.2,\ x_1=0.3$ and $x_2=0.4$

Then $y_0=f(0.2)=-0.28398668,\ y_1=f(0.3)=0.00660095$ and $y_2=f(0.4)=0.24842440$

We first determine the coefficient polynomial $L_0(x), L_1(x)$ and $L_2(x)$

$$\begin{aligned}L_0(x)&=\frac{(x-0.3)(x-0.4)}{(0.2-0.3)(0.2-0.4)}\\&=\frac{(x-0.3)(x-0.4)}{(-0.1)(-0.2)}\\&=\frac{(x-0.3)(x-0.4)}{0.02}\end{aligned}$$

$$\begin{aligned}L_1(x)&=\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\\&=\frac{(x-0.2)(x-0.4)}{(0.3-0.2)(0.3-0.4)}\\&=\frac{(x-0.2)(x-0.4)}{(0.1)(-0.1)}\\&=-\frac{(x-0.2)(x-0.4)}{0.01}\end{aligned}$$

and

$$\begin{aligned}L_2(x)&=\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\\&=\frac{(x-0.2)(x-0.3)}{(0.4-0.2)(0.4-0.3)}\\&=\frac{(x-0.2)(x-0.3)}{(0.2)(0.1)}\\&=\frac{(x-0.2)(x-0.3)}{0.02}\end{aligned}$$

Now the interpolate polynomial of degree two

$$\begin{aligned}p_2(x)&=L_0(x)\cdot y_0+L_1(x)\cdot y_1+L_2(x)\cdot y_2\\&=\frac{(x-0.3)(x-0.4)}{0.02}(-0.28398668)-\frac{(x-0.2)(x-0.4)}{0.01}(0.00660095)\\&\qquad+\frac{(x-0.2)(x-0.3)}{0.02}(0.24842440)\\&=-2.4382090x^2+4.12498080x-1.011454480\end{aligned}$$

And $p_2(0.25)=-2.4382090(0.25)^2+4.12498080(0.25)-1.011454480$ by using calculator

= $\boxed{-0.13259734}$

Comment

Step 8 of 11

To construct interpolation polynomial of degree three consider all the four nodal points

i.e. $x_0=0.2,\ x_1=0.3,\ x_2=0.4$ and $x_3=0.1$

Then $y_0=f(0.2)=-0.28398668,\ y_1=f(0.3)=0.00660095,\ y_2=f(0.4)=0.24842440$

and $y_3=f(0.1)=-0.62049958$

We first determine the coefficient polynomial $L_0(x), L_1(x), L_2(x)$ and $L_3(x)$

$$\begin{aligned}L_0(x)&=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\\&=\frac{(x-0.3)(x-0.4)(x-0.1)}{(0.2-0.3)(0.2-0.4)(0.2-0.1)}\\&=\frac{(x-0.3)(x-0.4)(x-0.1)}{(-0.1)(-0.2)(0.1)}\\&=\frac{(x-0.3)(x-0.4)(x-0.1)}{0.002}\end{aligned}$$

$$\begin{aligned}L_1(x)&=\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\\&=\frac{(x-0.2)(x-0.4)(x-0.1)}{(0.3-0.2)(0.3-0.4)(0.3-0.1)}\\&=\frac{(x-0.2)(x-0.4)(x-0.1)}{(0.1)(-0.1)(0.2)}\\&=-\frac{(x-0.2)(x-0.4)(x-0.1)}{0.002}\end{aligned}$$

$$= \frac{}{(0.4-0.2)(0.4-0.3)(0.4-0.1)}$$
$$= \frac{(x-0.2)(x-0.3)(x-0.1)}{(0.2)(0.1)(0.3)}$$
$$= \frac{(x-0.2)(x-0.3)(x-0.1)}{0.006}$$

and

$$L_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$
$$= \frac{(x-0.2)(x-0.3)(x-0.4)}{(0.1-0.2)(0.1-0.3)(0.1-0.4)}$$
$$= \frac{(x-0.2)(x-0.3)(x-0.4)}{(-0.1)(-0.2)(-0.3)}$$
$$= -\frac{(x-0.2)(x-0.3)(x-0.4)}{0.006}$$

Now the interpolate polynomial of degree three

$$p_3(x) = L_0(x)\cdot y_0 + L_1(x)\cdot y_1 + L_2(x)\cdot y_2 + L_3(x)\cdot y_3$$
$$= \frac{(x-0.3)(x-0.4)(x-0.1)}{0.002}(-0.28398668) - \frac{(x-0.2)(x-0.4)(x-0.1)}{0.002}(0.00660095)$$
$$+ \frac{(x-0.2)(x-0.3)(x-0.1)}{0.006}(0.24842440) - -\frac{(x-0.2)(x-0.3)(x-0.4)}{0.006}(-0.62049958)$$
$$= -0.4731516666x^3 - 2.0123725000x^2 + 4.0019613666x - 1.000098840$$

And

$$p_3(0.25) = -0.4731516666(0.25)^3 - 2.0123725000(0.25)^2 + 4.0019613666(0.25) - 1.000098840$$

by using calculator

= $\boxed{-0.13277477}$

Comment

Step 9 of 11

(d) We are given with $f(0.6) = -0.17694460$, $f(0.7) = 0.01375227$, $f(0.8) = 0.22363362$ and

$f(1.0) = 0.65809197$

To construct interpolation polynomial of degree one, consider only two nodal points

$x_0 = 0.8$ and $x_1 = 1.0$

Then $y_0 = f(0.8) = 0.22363362$ and $y_1 = f(1.0) = 0.65809197$

We first determine the coefficient polynomial $L_0(x)$ and $L_1(x)$

$$L_0(x) = \frac{x-x_1}{x_0-x_1}$$
$$= \frac{x-1}{0.8-1}$$
$$= -\frac{x-1}{0.2}$$

$$L_1(x) = \frac{x-x_0}{x_1-x_0}$$
$$= \frac{x-0.8}{1-0.8}$$
$$= \frac{x-0.8}{0.2}$$

Now the interpolate polynomial of degree one

$$p_1(x) = L_0(x)\cdot y_0 + L_1(x)\cdot y_1$$
$$= -\frac{x-1}{0.2}(0.22363362) + \frac{x-0.8}{0.2}(0.65809197)$$
$$= 2.172291750x - 1.514199780$$

And $p_1(0.9) = 2.172291750(0.9) - 1.514199780$ by using calculator

= $\boxed{0.440862795}$

Comment

Step 10 of 11

To construct interpolation polynomial of degree two consider all the three nodal points

i.e. $x_0 = 0.8,\ x_1 = 1.0$ and $x_2 = 0.7$

Then $y_0 = f(0.8) = 0.22363362$, $y_1 = f(1.0) = 0.65809197$ and $y_2 = f(0.7) = 0.01375227$

We first determine the coefficient polynomial $L_0(x), L_1(x)$ and $L_2(x)$

$$L_0(x) = \frac{(x-1)(x-0.7)}{(0.8-1)(0.8-0.7)}$$
$$= \frac{(x-1)(x-0.7)}{(-0.2)(0.1)}$$
$$= -\frac{(x-1)(x-0.7)}{0.02}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$
$$= \frac{(x-0.8)(x-0.7)}{(1-0.8)(1-0.7)}$$
$$= \frac{(x-0.8)(x-0.7)}{(0.2)(0.3)}$$
$$= \frac{(x-0.8)(x-0.7)}{0.06}$$

and

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$
$$= \frac{(x-0.8)(x-1)}{(0.7-0.8)(0.7-1)}$$
$$= \frac{(x-0.8)(x-1)}{(-0.1)(-0.3)}$$
$$= \frac{(x-0.8)(x-1)}{0.03}$$

Now the interpolate polynomial of degree two

$$p_2(x) = L_0(x)\cdot y_0 + L_1(x)\cdot y_1 + L_2(x)\cdot y_2$$
$$= -\frac{(x-1)(x-0.7)}{0.02}(0.22363362) + \frac{(x-0.8)(x-0.7)}{0.06}(0.65809197)$$
$$+ \frac{(x-0.8)(x-1)}{0.03}(0.01375227)$$
$$= 0.24492750x^2 + 1.73142224999x - 1.3182577799$$

And $p_2(0.9) = 0.24492750(0.9)^2 + 1.73142224999(0.9) - 1.3182577799$ by using calculator

= $\boxed{0.438413520}$

Comment

Step 11 of 11

To construct interpolation polynomial of degree three consider all the four nodal points

i.e. $x_0 = 0.8,\ x_1 = 1.0,\ x_2 = 0.7$ and $x_3 = 0.6$

Then $y_0 = f(0.8) = 0.22363362$, $y_1 = f(1.0) = 0.65809197$ $y_2 = f(0.7) = 0.01375227$

and $y_3 = f(0.6) = -0.17694460$

We first determine the coefficient polynomial $L_0(x), L_1(x), L_2(x)$ and $L_3(x)$

$$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$
$$= \frac{(x-1)(x-0.7)(x-0.6)}{(0.8-1)(0.8-0.7)(0.8-0.6)}$$
$$= \frac{(x-1)(x-0.7)(x-0.6)}{(-0.2)(0.1)(0.2)}$$
$$= -\frac{(x-1)(x-0.7)(x-0.6)}{0.002}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$
$$= \frac{(x-0.8)(x-0.7)(x-0.6)}{(1-0.8)(1-0.7)(1-0.6)}$$
$$= \frac{(x-0.8)(x-0.7)(x-0.6)}{(0.2)(0.3)(0.4)}$$
$$= \frac{(x-0.2)(x-0.3)(x-0.1)}{0.024}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$$
$$= \frac{(x-0.8)(x-1)(x-0.6)}{(0.7-0.8)(0.7-1)(0.7-0.6)}$$
$$= \frac{(x-0.8)(x-1)(x-0.6)}{(-0.1)(-0.3)(0.1)}$$
$$= \frac{(x-0.8)(x-1)(x-0.6)}{0.003}$$

and

$$L_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$
$$= \frac{(x-0.8)(x-1)(x-0.7)}{(0.6-0.8)(0.6-1)(0.6-0.7)}$$
$$= \frac{(x-0.8)(x-1)(x-0.7)}{(-0.2)(-0.4)(-0.1)}$$
$$= -\frac{(x-0.8)(x-1)(x-0.7)}{0.002}$$

Now the interpolate polynomial of degree three

$$p_3(x) = L_0(x)\cdot y_0 + L_1(x)\cdot y_1 + L_2(x)\cdot y_2 + L_3(x)\cdot y_3$$
$$= -\frac{(x-1)(x-0.7)(x-0.6)}{0.002}(0.22363362) + \frac{(x-0.2)(x-0.3)(x-0.1)}{0.024}(0.65809197)$$
$$+ \frac{(x-0.8)(x-1)(x-0.6)}{0.003}(0.01375227) - \frac{(x-0.8)(x-1)(x-0.7)}{0.002}(-0.17694460)$$
$$= -1.7857412500x^3 + 4.7092806250x^2 - 1.9472047250x - 0.31824267999$$

And

$$p_3(0.9) = -1.7857412500(0.9)^3 + 4.7092806250(0.9)^2 - 1.9472047250(0.9) - 0.31824267999$$

by using calculator

= $\boxed{0.4419850025}$

Comment

Recommended solutions for you in Chapter 3.1